Heat Transfer

Unit I
Introduction and Heat Conduction

Code	Subject	Teaching Scheme Hrs / week			Examination Scheme					Total Marks	Credits	
		Lecture	Tut	Pract	In-Sem	ESE	TW	PR	OR		Th	TW / PR / OR
302041	Design of Machine Elements-I	4	-	2	30@	70@	50	-		150	4	1
302042	Heat Transfer*	4	-	2	30	70		50	-	150	4	1
302043	Theory of Machines-II$	3	1		30	70	25	-	25	150	3	1
302044	Turbo Machines	3	-	2	30	70	-	-	25	125	3	1
302045	Metrology and Quality Control$	3	-	2	30	70	-	-	25	125	3	1
302046	Skill Development	-	-	2	-	-	25	25	-	50	-	1
Total		17	1	10	150	350	100	75	75	750	17	6
											23	

Course Objectives:

- 1.Identify the important modes of heat transfer and their applications.
- 2.Formulate and apply the general three dimensional heat conduction equations.
- 3.Analyze the thermal systems with internal heat generation and lumped heat capacitance.
- 4.Understand the mechanism of convective heat transfer
- 5.Determine the radiative heat transfer between surfaces.
- 6.Describe the various two phase heat transfer phenomenon. Execute the effectiveness and rating of heat exchangers.

Course Outcomes:

- CO 1: Analyze the various modes of heat transfer and implement the basic heat conduction equations for steady one dimensional thermal system.
- CO 2: Implement the general heat conduction equation to thermal systems with and without internal heat generation and transient heat conduction.
- CO 3:Analyze the heat transfer rate in natural and forced convection and evaluate through experimentation investigation.
- CO 4:Interpret heat transfer by radiation between objects with simple geometries.
- CO 5:Analyze the heat transfer equipment and investigate the performance.

Syllabus: UNIT 1: (10 hrs)

- **Introduction and Basic Concepts**:
  - Application areas of heat transfer, Modes and Laws of heat transfer, Three dimensional heat conduction equation in Cartesian coordinates and its simplified equations, thermal conductivity, Thermal diffusivity, Thermal contact Resistance Boundary and initial conditions: Temperature boundary condition, heat flux boundary condition, convection boundary condition, radiation boundary condition.
- **One dimensional steady state heat conduction without heat generation:**
  - Heat conduction in plane wall, composite slab, composite cylinder, composite sphere, electrical analogy, concept of thermal resistance and conductance, three dimensional heat conduction equations in cylindrical and spherical coordinates (no derivation) and its reduction to one dimensional form, critical radius of insulation for cylinders and spheres, economic thickness of insulation

Prerequisites

Thermodynamics, Fluid Mechanics

References

Incropera FP and Dewitt DP, *Fundamentals of Heat and Mass Transfer*, Fifth edition, John Wiley and Sons, 2010.

Cengel YA, *Heat and Mass Transfer - A Practical Approach*, Third edition, McGraw-Hill, 2010.

Holman JP, *Heat Transfer*, McGraw-Hill, 1997.

Introduction:

- What, How, and Where?
  Thermodynamics and Heat transfer Application
  Physical mechanism of heat transfer
  Conduction:
- Introduction 1D, steady-state 2D, steady-state
  Transient

Convection:

- Introduction External and internal flows
  Free convection Boiling and condensation Heat exchangers

Radiation:

- Introduction View factors

Fourier's Law of Heat Conduction

Rate of heat transfer by conduction (through a solid) in a given direction is proportional to the area normal to the direction of heat flow and the temp gradient in that direction. Mathematically ;

$$Q \propto A \frac{\Delta T}{\Delta x} \; Watt$$

$$Q = -kA \frac{dT}{dx} \; Watt \, (J / s)$$

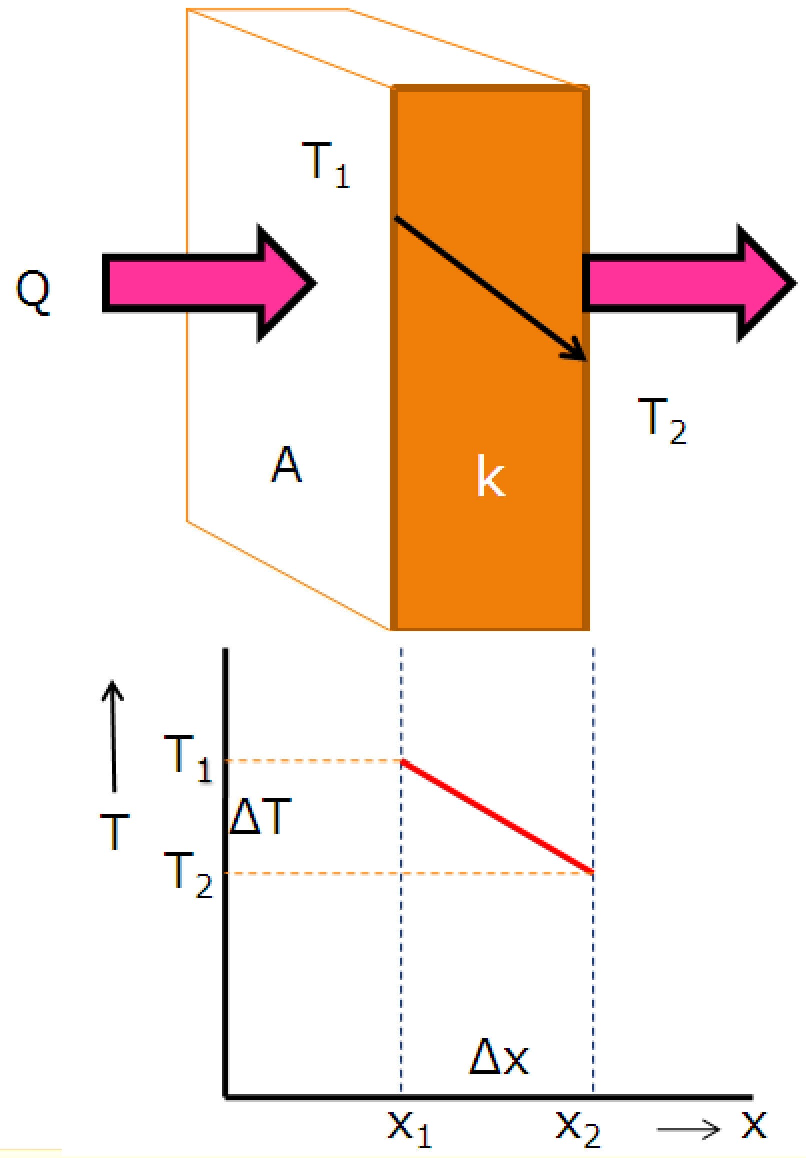
Conduction

$$Q = -kA\frac{dT}{dx}$$

$$Q = kA\frac{(T_1 - T_2)}{(x_1 - x_2)}$$

$$Q = -kA\frac{(T_1 - T_2)}{(x_2 - x_1)}$$

$$Q = kA\frac{\Delta T}{\Delta x}$$



Heat Flux $q = Q/A$ W/m^2

Assumptions of Fourier's Law

1. Unidirectional heat flow (only one direction)
2. Steady state heat flow
3. Constant temp gradient
4. Constant conductivity, k
5. Both faces isothermal

Variation of Thermal Conductivity

1. It is the property of material; defined as ability of material to conduct heat through it.

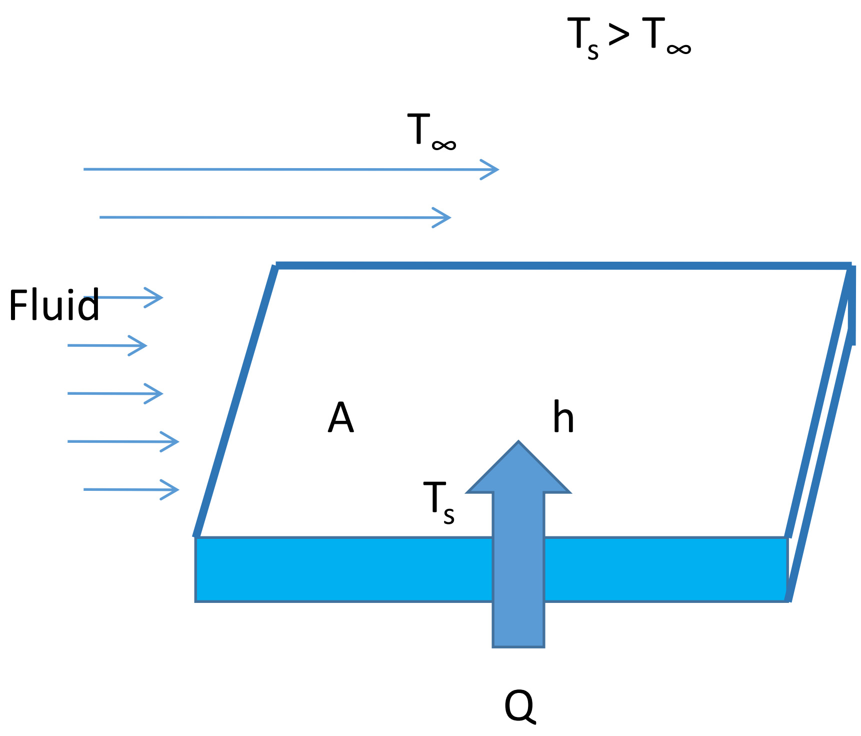
2. Thermal conductivity in decreasing order :
   Metals » Non-metallic Solids » Liquids » Gases

3. Higher conductivity in metals due to free electrons in their outer orbits

4. k depends on grain structure. When k is different in different directions (k_x , k_y , k_z), material is known as anisotropic. When k is constant in all directions, it is called Isotropic.

5. k is strongly dependent on temp; $k=k_o(1+\alpha T)$

Convection



$$Q = hA(T_s - T_\infty); \text{ Watt}$$

Heat Radiation

All bodies continuously emit energy if their temp is above zero absolute (0K) and energy thus emitted is called thermal radiation.

Thermal radiations are electromagnetic waves and do not require any medium for propagation.

Thermal radiation is a surface phenomenon.

Theories of Thermal Radiation

- Wave/Maxwell's Classical Theory : Propagation by electromagnetic waves
- Quantum/ Planck's Theory: Propagation by quanta possessing certain amount of energy

Stefan Boltzmann's Law of Radiation

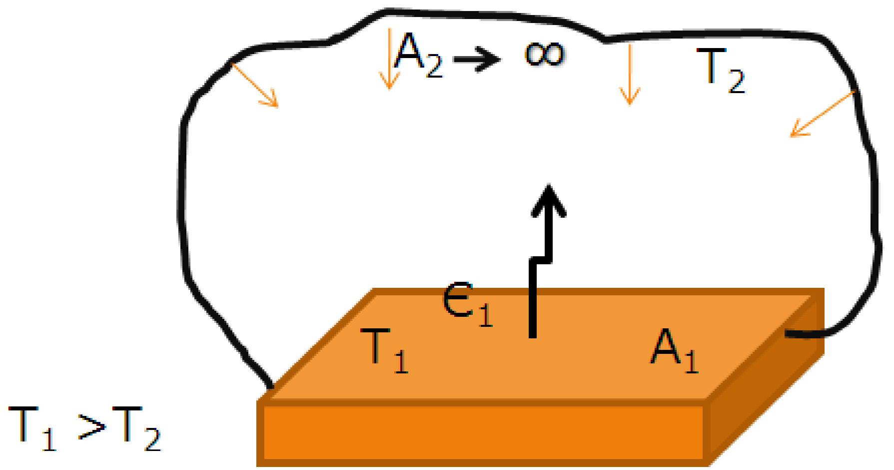
Thermal radiation emitted by a black body is proportional to the Fourth Power of its absolute temp.

Mathematically;

$q \propto T^4$ W/m²;

$Q = \sigma A T^4$ W; where σ is Stefan Boltzmann's constant (5.67 x 10^{-8} W/m²K⁴)

$$Q = A_1 \epsilon_1 \sigma (T_1^4 - T_2^4)$$



Examples of Composite Structures

- Walls of buildings
- Walls of home refrigerators
- Insulated pipe carrying steam
- Walls of a furnace
- Walls of a cold storage
- Hot case for food

Conduction HT

We need to determine $T(x, y, z, t)$

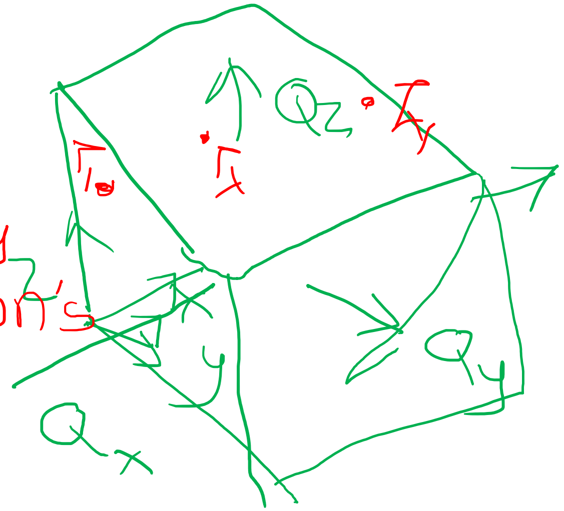
It depends on –

- BC's – Boundary Condition's
- IC's
- properties
- Geometry

3D – 6 BC's

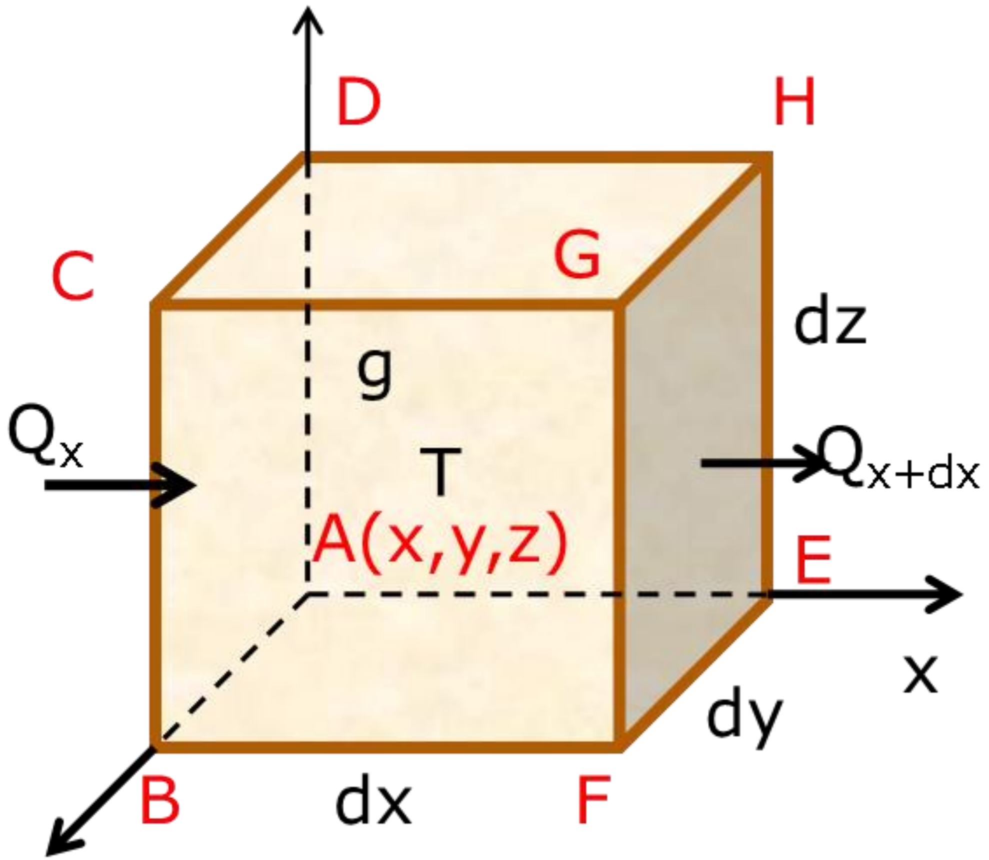
2D – 4 BC's

1D – 2 BC's



BC → $x = 0$: $T = T_1$

General Heat Conduction Equation In Cartesian Coordinates



As per Fourier's Law,

heat entering $dQ_x = -k_x(\delta y \delta z)\partial T/\partial x$

Similarly, heat leaving,

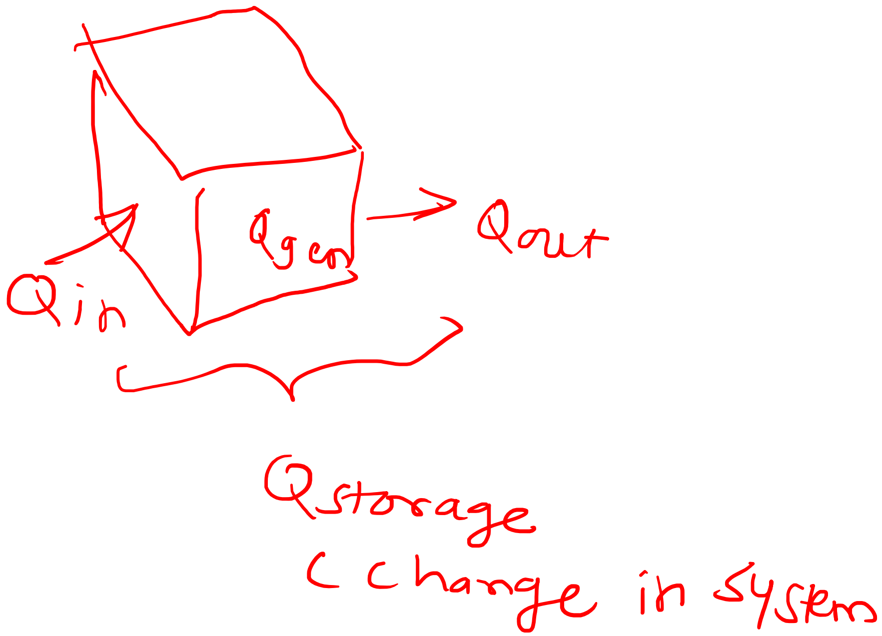
$$dQ_{x+\delta x} = dQ_x + \partial/\partial x(dQ_x)\delta x$$

So, net heat flow into the element in x-direction/time;

$$dQ_x - dQ_{x+dx} = -\frac{\partial}{\partial x}(dQ_x)\delta x$$

$$= -\frac{\partial}{\partial x}\left(-k_x \delta y \delta z . \frac{\partial T}{\partial x}\right)\delta x$$

$$= \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)\delta x \delta y \delta z$$



Thus, net heat flow in to the element from all directions by conduction in certain time δt will be:

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)\right]\delta x\,\delta y\,\delta z\,\delta t$$

Now, internal heat generation in time $\delta t = g.\delta x \delta y \delta z \delta t$

Heat gain by the element from above, will result in energy storage and will increase its temp.

Let δT be the rise in temp in time δt, the net heat storage in the element in time δt ;

$$(mC_p\ \Delta T) = \rho V C_p\ \delta T$$

$$= \rho C_p\ \delta T\ \delta x \delta y \delta z$$

<u>Energy Balance Equation:</u>

Net heat conducted in to the element from all Directions +Heat generated within the element = Energy stored in the element

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)\right]\delta x \delta y \delta z \delta t$$

$$+ g.\delta x \delta y \delta z \delta t \quad = \quad \rho C_p \delta T.\delta x \delta y \delta z$$

Dividing the Equation by δxδyδzδt, we get;

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)+g=\rho C_p \frac{\partial T}{\partial t}$$

For isotropic material, $k_x=k_y=k_z=k$ constant

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\cdots \text{Where } \alpha \text{ is thermal diffusivity} = \frac{k}{\rho C_p}\ m^2/s$$

Fourier's Equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Poisson's Equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g}{k} = 0$$

Laplace Equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

Steady State, One Dimensional Equation w/o g:

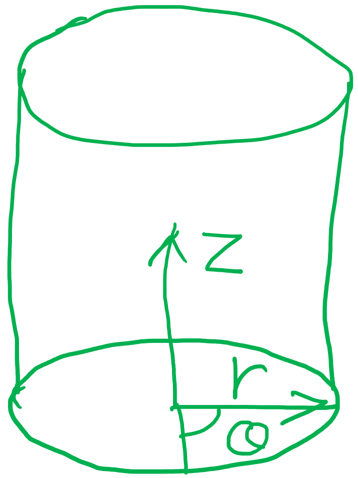
$$\frac{d^2 T}{dx^2} = o$$

General Heat Conduction Equation In Cylindrical Coordinates

By substituting x=r.cosθ; y=r.sinθ and z=z, we get General Heat Conduction Equation in Polar/ Cylindrical Coordinates:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\cdots For\ isotropic\ material\ with\quad k = constt$$



Poisson's Equation: $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{g}{k}=0$$

Radial heat conduction w/o g: $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

radiirn

Laplace form

2nd order eqn

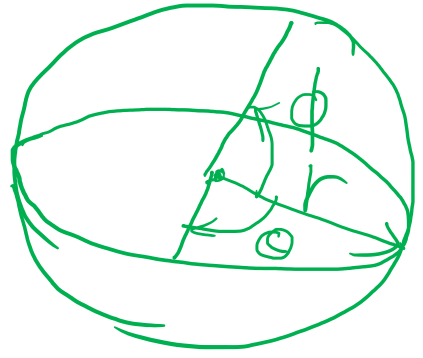
2-BC's

BC's: r = 0 : T = T_1

r = r : T = T_2

General Heat Conduction Equation In Spherical Coordinates

Similarly, by substituting x=r.sinθ.cosΦ; y= r.sinθsinΦ and z=r.sinθ, we get heat conduction equation in Spherical Coordinates:



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

For isotropic material with k = constt

Poisson's Equation:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{g}{k}=0$$

Radial heat conduction w/o g:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$

BVP — 2 BC's ⇒ $r = r_1$ $T = T_1$; $r = r_2$ $T = T_2$

IBVP — IC's = time

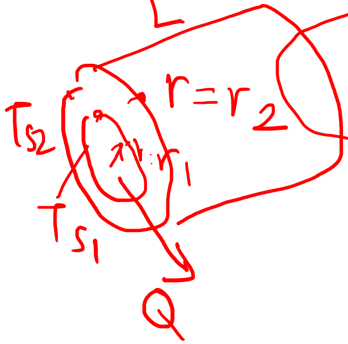
Insulating Materials

- Materials which are used to reduce the heat transfer rate from / to the system, are known as INSULATOTRS
- Examples are glass wool, plastics, wood, brick, cement, asbestos, rubber, grass, saw dust, cork, glass, clay, etc
- Insulators have low conductivity (generally k <2 W/mK)
- Insulating materials should be cheaper, able to withstand higher temp and humidity, should remain in applied shape and have long life, odorless, non-reactive,
- Practical applications are in refrigerators & air conditioners, buildings, conduits carrying high temp fluids like steam/ chemicals, plastic handles of kitchen utensils, furnaces, cold storages, offices etc

Conductivities of some Insulating Materials

Materials	Conductivity k (W/mK)
Wood	1.2 – 0.8
Brick	0.9 – 1.3
Concrete	0.8 – 0.9
Glass	0.7 – 0.8
Asbestos	0.2 – 0.4
Glass fiber	0.04
Cork	0.03
Plastics	0.9 – 0.04
Air	0.02
Clay	1.02
Gypsum	0.3
Saw Dust	0.07

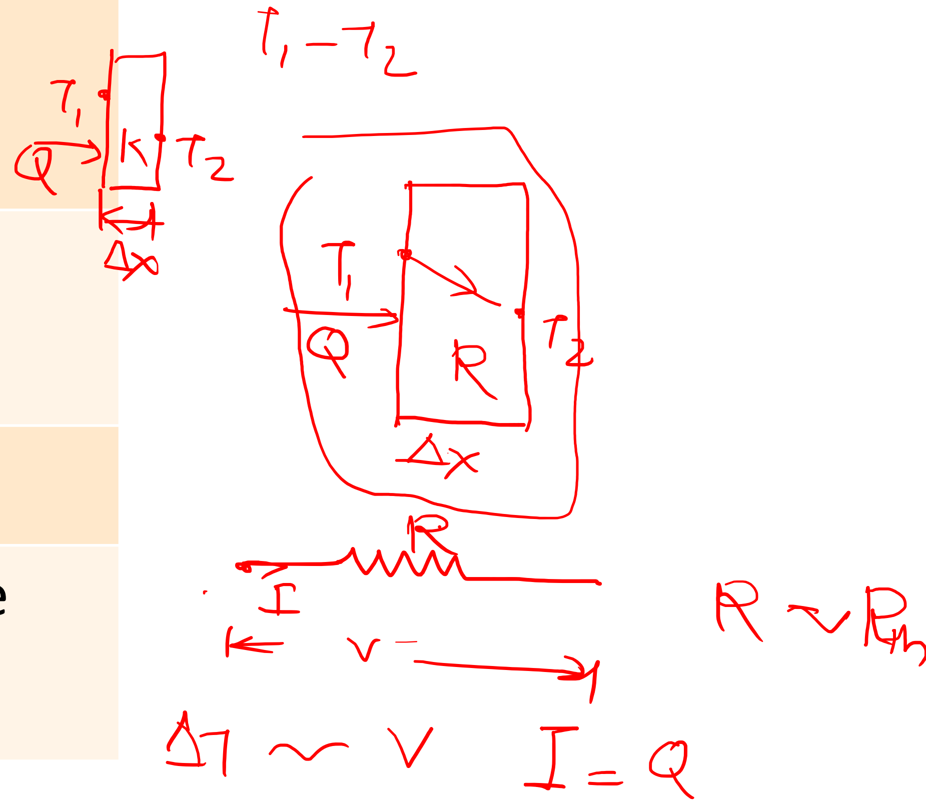
	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,cond}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$



[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

Electrical Analogy

	Electrical Energy	Heat Energy
What flows?	Electrons	Heat energy through electrons
Driving Potential	Voltage Diff, ΔV	Temp Diff, ΔT
Flow	Current, I	Heat Transfer Rate, Q
Resistance to flow	ρ, A, L of conductor	R, Thermal Resistance



Electrical Analogy

As per Ohm's Law, I = ΔV/R

$V = IR$

Similarly, Heat Flow Rate, Q = ΔT/R = C.ΔT;
where R is thermal resistance & 1/R=C conductance

$R_{th} = \frac{\Delta x}{kA}$

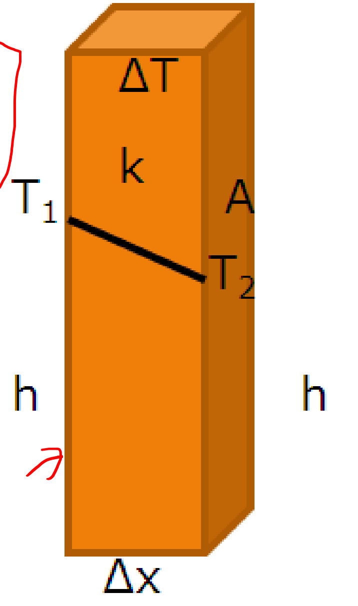
$\Delta T = Q \cdot R_{th}$

$Q = \frac{\Delta T}{R_{th}} \sim I = \frac{V}{R}$

Conductive Resistance:

$$Q = kA\frac{\Delta T}{\Delta x} = \frac{\Delta T}{\frac{\Delta x}{kA}} = \frac{\Delta T}{R}$$

$Q = kA \cdot \frac{\Delta T}{\Delta x} = \frac{\Delta T}{\left(\frac{\Delta x}{kA}\right)} \rightarrow R_{th}$



$$Hence, R_{conductive} = \frac{\Delta x}{kA}$$

$R_{cond} = \frac{\Delta x}{kA}$

Electrical Analogy

Convective Resistance:

Newtons law of cooling

$$Q = hA(T_2 - T_\infty) = \frac{\Delta T}{\frac{1}{hA}} = \frac{\Delta T}{R_{conv}}$$

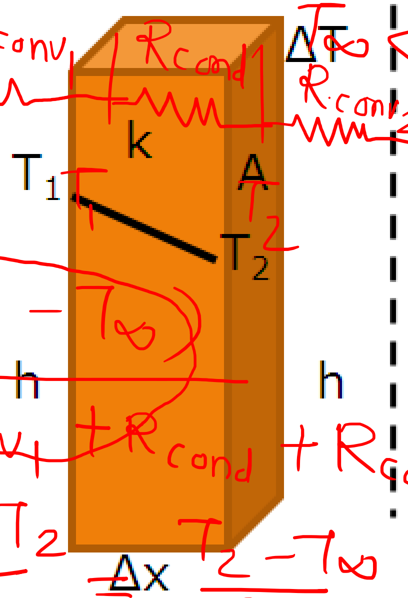
$$Q = h_i A (T_{f_1} - T_1)$$

$$Hence, R_{convective} = \frac{1}{hA}$$

$T_{f_1} > T_1$

$$Q = \frac{\Delta T_{total}}{R_{total}} = \frac{\Delta T_1}{R_1} = \frac{\Delta T_2}{R_2}$$

$$Q = \frac{\Delta T_{total}}{R_{total}} = \frac{(T_{f_1} - T_\infty)}{R_{conv_1} + R_{cond} + R_{conv_2}} = \frac{T_{f_1} - T_1}{R_{conv_1}} = \frac{T_1 - T_2}{R_{cond}} = \frac{T_2 - T_\infty}{R_{conv_2}}$$



Heat Transfer In Composite Structures

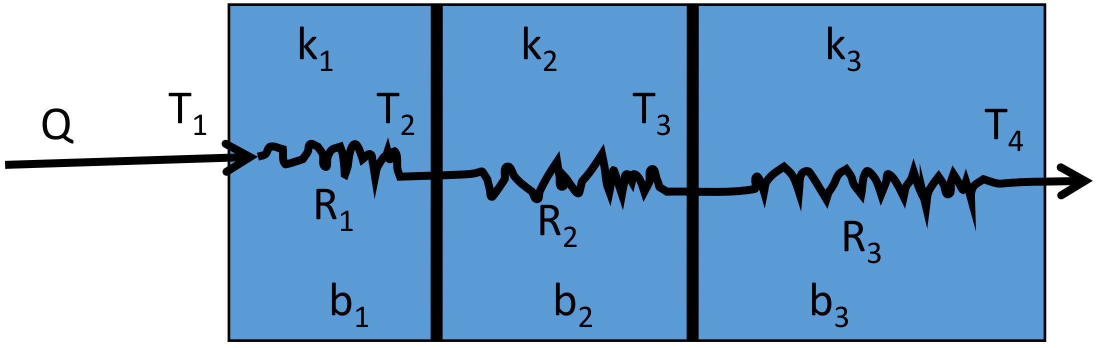
Resistance In Series

$Q = \Delta T / R$

$= (T_1 - T_2)/R_1$

$= (T_2 - T_3)/R_2$

$= (T_3 - T_4)/R_3$



On adding up;

$T_1 - T_4 = Q(R_1+R_2+R_3)$ OR $Q = (T_1 - T_4)/(R_1+R_2+R_3)$

$Q = \Delta T/R$; hence $R = R_1+R_2+R_3$

$R_1 = b_1/k_1A$; $R_2 = b_2/k_2A$; $R_3 = b_3/k_3A$

Heat Transfer In Composite Structures

Resistance In Parallel

$Q_1 = (T_1 - T_2)/R_1$

$Q_2 = (T_1 - T_2)/R_2$

$Q_3 = (T_1 - T_2)/R_3$

On adding;

$Q = Q_1 + Q_2 + Q_3$

$= (T_1 - T_2) * (1/R_1 + 1/R_2 + 1/R_3)$

$= \Delta T * 1/R;$

Hence $1/R = 1/R_1 + 1/R_2 + 1/R_3$

$Q = kA \frac{dT}{dx}$

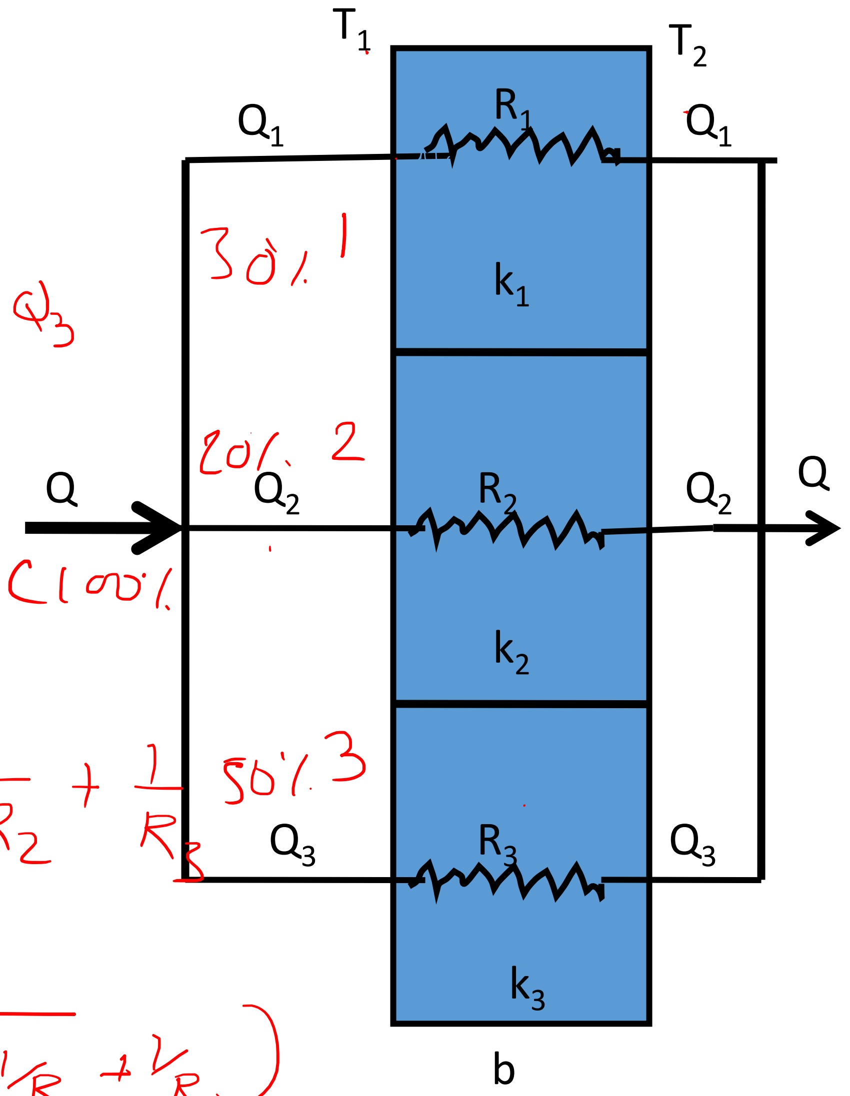
$R_1 = \frac{L}{K_1 A_1}$

$Q_1 = Q_2 = Q_3$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

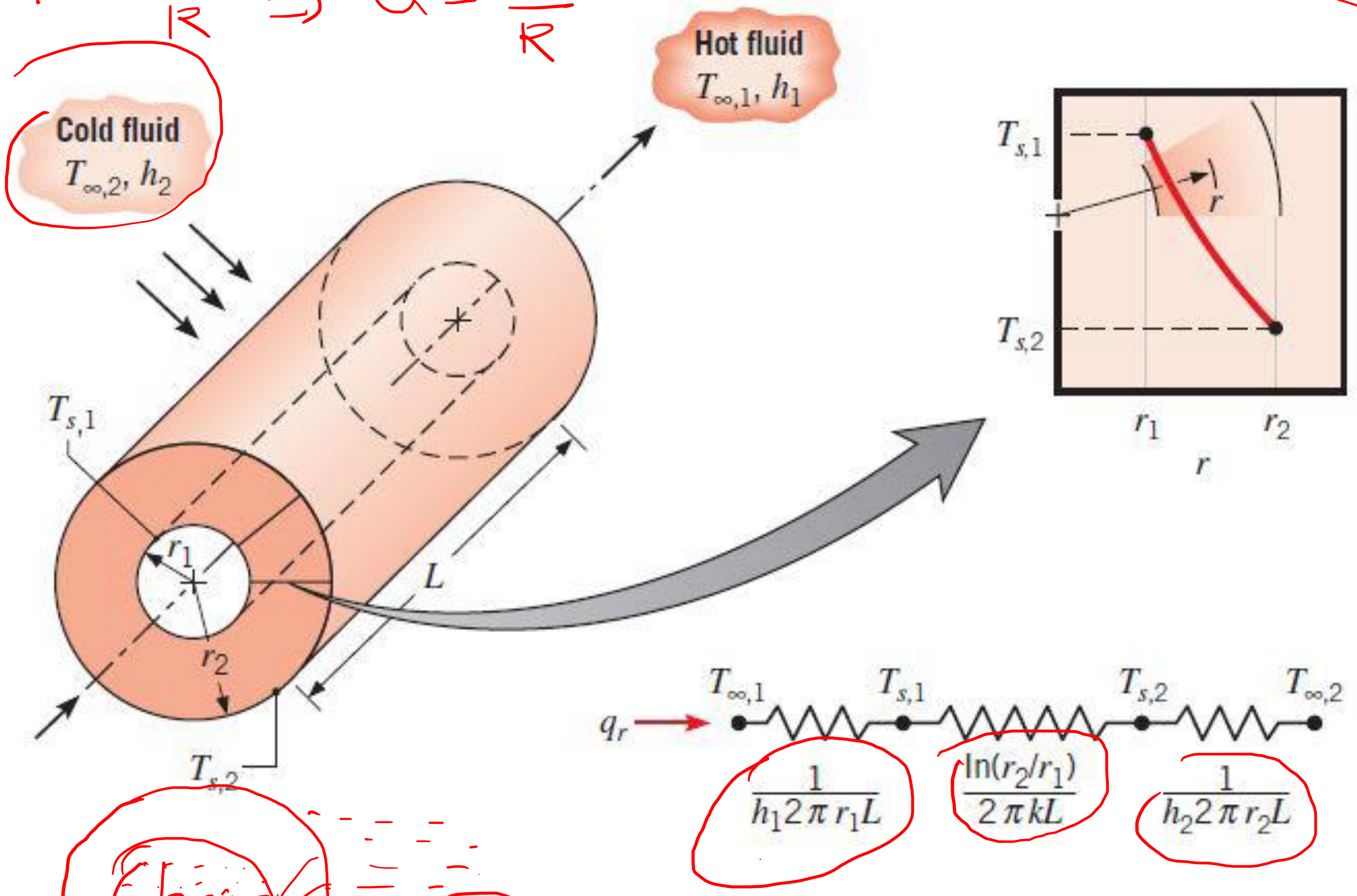
$Q = \frac{\Delta T}{R}$

$R = \left(\frac{1}{1/R_1 + 1/R_2 + 1/R_3}\right)$



$I = \frac{V}{R} \Rightarrow Q = \frac{\Delta T}{R}$

$Q_{cond} = 2\pi L k \frac{\Delta T}{\ln(r_2/r_1)}$



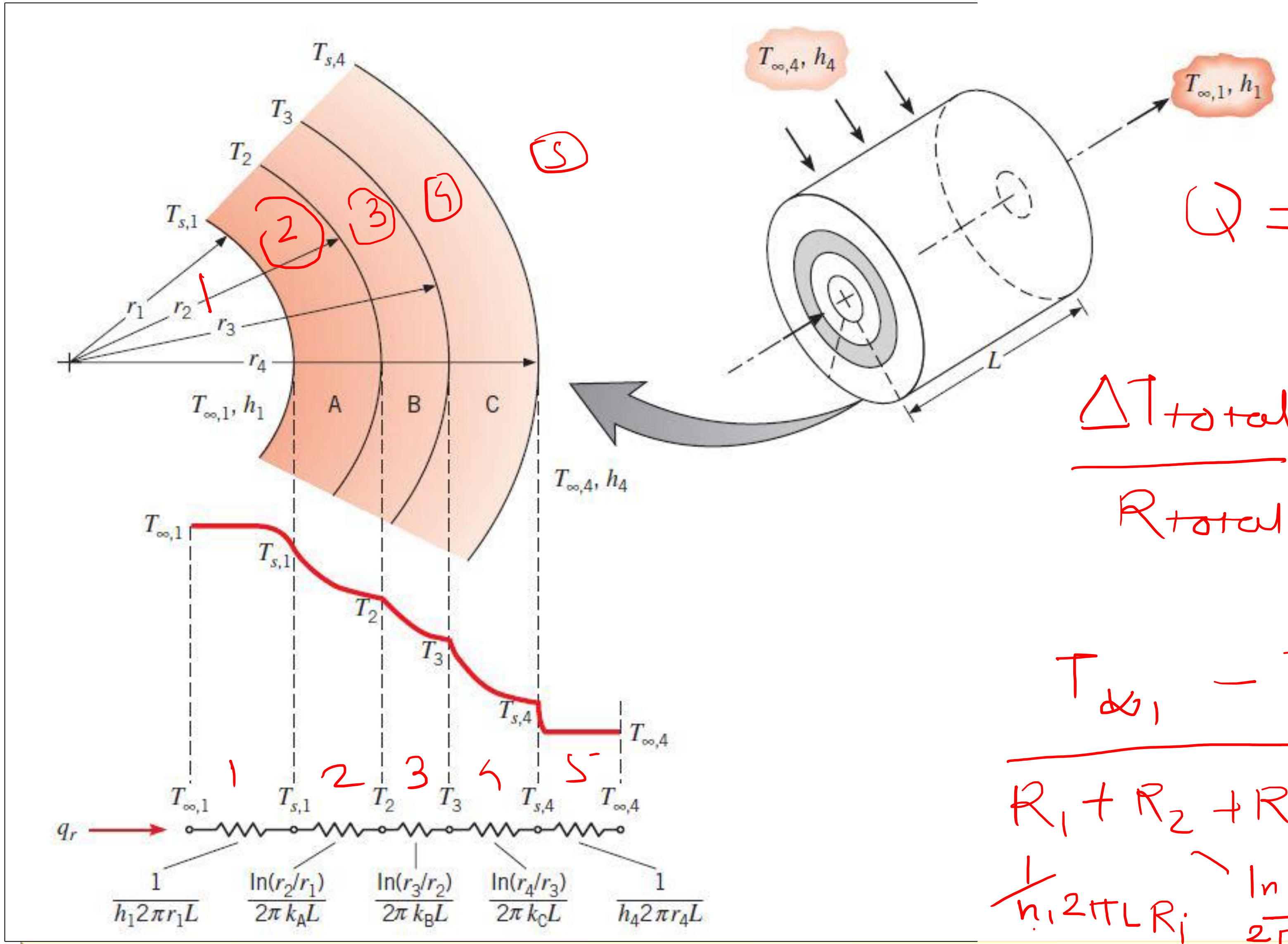
$A_i = 2\pi r_i L$

$A_o = 2\pi r_o L$

$Q_{conv} = h_1 A_i (T_{\infty_1} - T_{s_1})$

$R_i \Rightarrow \frac{1}{h_1 A_i}$

$Q_{conv} + Q_{cond} + Q_{convout} = Q_{radial}$



$$Q = Q_{conv1} + Q_{cond_A} + Q_{cond_B} + Q_{cond_C} + Q_{conv\ outside}$$

$$\frac{\Delta T_{total}}{R_{total}} = \frac{\Delta T_1}{R_1} = \frac{\Delta T_2}{R_2} = \frac{\Delta T_3}{R_3} = \frac{\Delta T_5}{R_5}$$

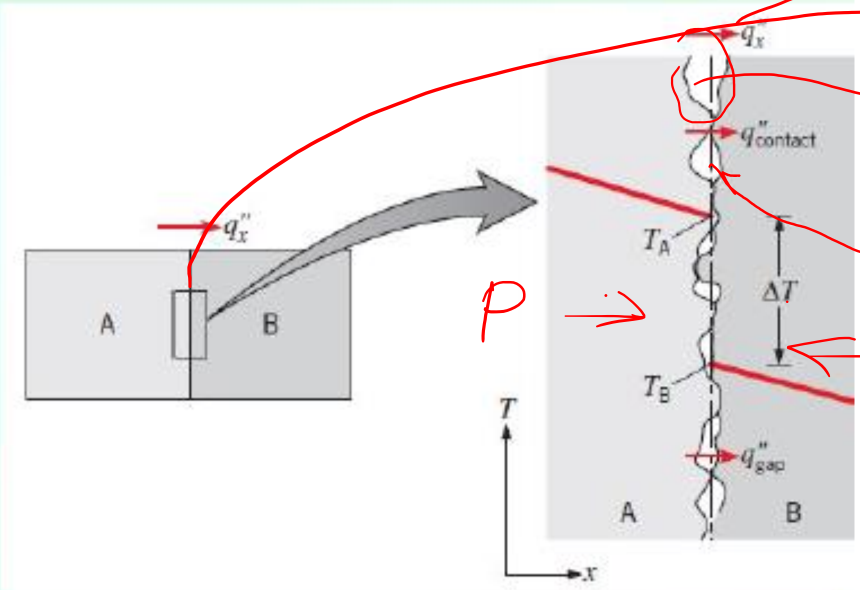
$$\frac{T_{\infty 1} - T_{\infty 2}}{R_1 + R_2 + R_3 + R_4 + R_5}$$

$$R_1 = \frac{1}{h_1 2\pi L R_i}, \quad R_2 = \frac{\ln(r_2/r_1)}{2\pi L K_A}, \quad R_3 = \frac{\ln(r_3/r_2)}{2\pi L K_B}, \quad R_4 = \frac{\ln(r_4/r_3)}{2\pi L K_C}, \quad R_5 = \frac{1}{h_o 2\pi L r_4}$$

Contact Resistance

$$R''_{t,c} = \frac{T_A - T_B}{q''_x}$$

$$q = \frac{\Delta T}{R_{tc}}$$



100% → 10% loss by contact

air packet

Smooth surface $T_A = T_B$

$T_A - T_B$

$T_A \neq T_B$

$$Q = \frac{\Delta T}{R_1 + R_2 + R_{contact}}$$

Thermal Resistance, $R''_{t,c} \times 10^4$ $(m^2 \cdot K/W)$

(a) Vacuum Interface			(b) Interfacial Fluid	
Contact pressure	100 kN/m²	10,000 kN/m²	Air	2.75
Stainless steel	6–25	0.7–4.0	Helium	1.05
Copper	1–10	0.1–0.5	Hydrogen	0.720
Magnesium	1.5–3.5	0.2–0.4	Silicone oil	0.525
Aluminum	1.5–5.0	0.2–0.4	Glycerine	0.265

Unsteady State Heat Transfer

- Whenever a heat transfer system is switched on/ started, it takes some time to attain steady value of heat transfer rate. Heat transfer rate under these conditions keeps varying with passage of time. This heat transfer system is said to be transferring heat under unsteady state / transient conditions. Here, temperature also keeps varying at various locations in the system with time. Hence, temp is a function of both location as well as time.
- Similar situation occurs when a heat transfer system is switched off / shut off, but in reverse direction
- Examples are starting/firing of a furnace, heating of a body, switching on a heater, starting of an engine, etc

- Whenever a heat transfer system is switched on/ started, it takes some time to stabilize the heat transfer rate when it becomes constant and does not change with time. This heat transfer system is said to be transferring heat under steady state conditions. Here, temperatures attain constant values at various locations in the system and do not vary with time. Hence, temp is a function of only location and not of time.
- Heat transfer rate is directly proportional to temp difference. Since temps attain constant values, temp difference also become constant hence heat transfer rate attains steady value
- This implies that whatever amount of heat energy is being received by the system, at same rate it is transferring out.
- This means that under steady state, system transfers / receives constant amount of heat energy per unit time

Types of Problems In Heat Transfer

1. Plate/Slab/Wall
2. Tube/Pipe/Cylinder
3. Sphere

- To increase Heat Transfer Rate
- To decrease Heat Transfer Rate

Thermal Diffusivity

Thermal Diffusivity is the ratio of thermal conductivity to heat storage capacity of the material.

$$\text{Denoted by } \alpha, \text{it is defined as}: \quad \alpha = \frac{k}{\rho C_p} \quad m^2/s$$

Larger the value of α, faster shall be the heat diffusion through the material.

Steady state heat conduction does not contain α, hence temp distr through material is determined by k only, where as in unsteady state heat conduction, temp distr is determined by α. (Both by k & ρC_p)

Example: Cooking steel utensils having copper bottom

One Dimensional Steady State Heat Conduction through Slab/Plane Wall

Consider a plane wall of thickness Δx of material having conductivity k with its faces maintained at temp T_1 & T_2

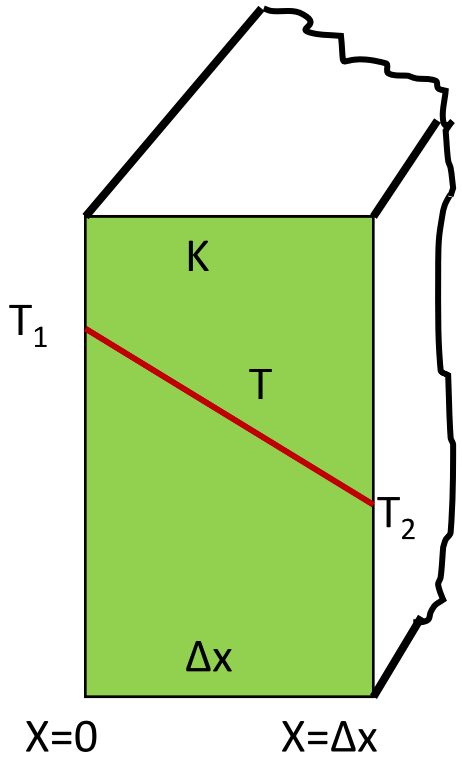
Steady state, one dimensional Heat conduction eqn will be:

$$\frac{d^2T}{dx^2} = 0$$

Integrating this equation twice;

$$We\ have\quad \frac{dT}{dx} = C_1 ....(1)\ Slope\ of\ Temp\ Profile$$

$$and\quad T = C_1 x + C_2 ....(2)\ Temp\ Profile$$



Heat Conduction through Slab/Plane Wall

Boundary Conditions:

1) At $x=0$; $T=T_1$
2) At $x=\Delta x$; $T=T_2$

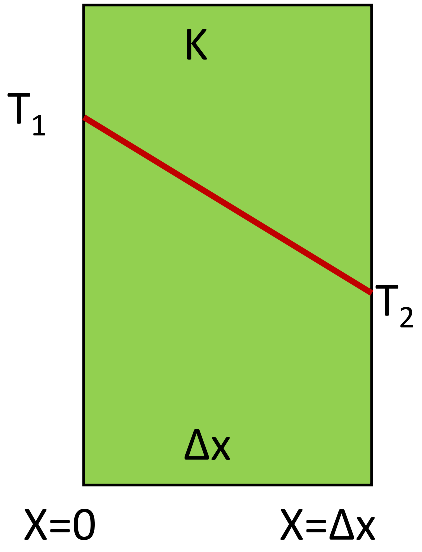
Applying BC 1), we get $T_1=C_1.0+C_2$

Hence $C_2=T_1$

Applying BC 2), we get $T_2=C_1.\Delta x+C_2$

Or $T_2=C_1.\Delta x+T_1$

$$\Rightarrow C_1 = \frac{T_2 - T_1}{\Delta x}$$



Substituting C_1 *and* C_2 *in Eqn..(2)*

$$We\ get \quad T = \frac{T_2 - T_1}{\Delta x}.x + T_1 \text{..........Temp Distribution}$$

$$Heat\ Flow Rate \quad Q = -kA\frac{dT}{dx}$$

fixed

$$Q = -kA\frac{T_2 - T_1}{\Delta x}$$

$$Q = kA\frac{(T_1 - T_2)}{\Delta x}$$

$$\frac{dT}{dx} =$$

Heat flux $= \frac{Q}{A} = k\frac{\Delta T}{\Delta x}$

$Q'' = q''$

$$\frac{W}{m^{\circ}C} \quad \frac{m^2({}^{\circ}C)}{m}$$

$$q''' = \frac{Q}{V}$$

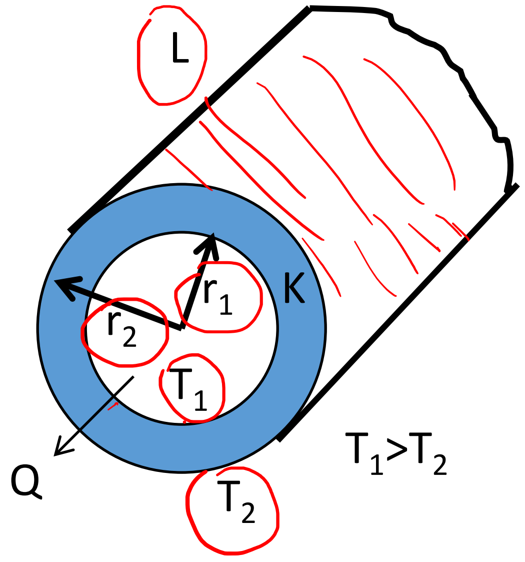
One Dimensional (Radial) Steady State Heat Conduction through Hollow Cylinder

Consider a hollow cylinder of inner radius r_1 and outer r_2 of length L of a material having conductivity k.

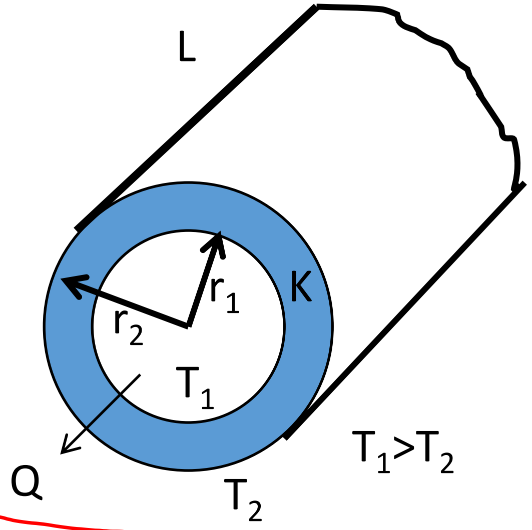
Inner surface of cylinder is at temp T_1 and outer at T_2

Conduction Equation for one dimensional (radial) Heat flow (without g) will be:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$



One Dimensional Steady State Heat Conduction through Hollow Cylinder



Integrating Equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

$$We\ have\quad \int\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

$$\Rightarrow r\frac{dT}{dr}=C_1 \quad or \quad \frac{dT}{dr}=\frac{C_1}{r}\ ....(1)$$

$$T(r)=C_1\ln r+C_2$$

On further Integration;

$$We\ have\quad T=C_1\ln r+C_2\ ....(2)$$

Heat Conduction through Hollow Cylinder

Boundary Conditions:

Eqn (2) $T = C_1.\ln r + C_2$

1) At $r = r_1$; $T = T_1$
2) At $r = r_2$; $T = T_2$

Substituting in Eqn ….(2); We have

$T_1 = C_1.\ln r_1 + C_2$ …..(3)

$T_2 = C_1.\ln r_2 + C_2$ ….(4)

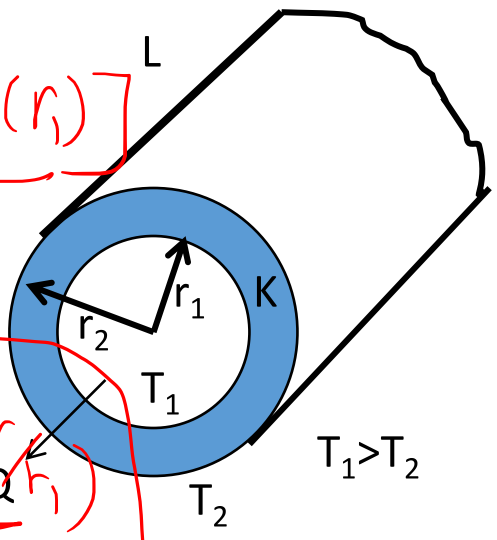
Subtracting eqn (4) from (3) and further substitution;

$$C_1 = \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} \quad and \; C_2 = T_1 - \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} . \ln r_1$$

$$T(r) = \frac{T_2 - T_1}{\ln(r_2/r_1)} . \ln(r) + T_1 - \frac{T_2 - T_1}{\ln(r_2/r_1)} . \ln(r_1)$$

$$(T(r) - T_1) = \frac{(T_2 - T_1)}{\ln(r_2/r_1)} [\ln(r) - \ln(r_1)]$$

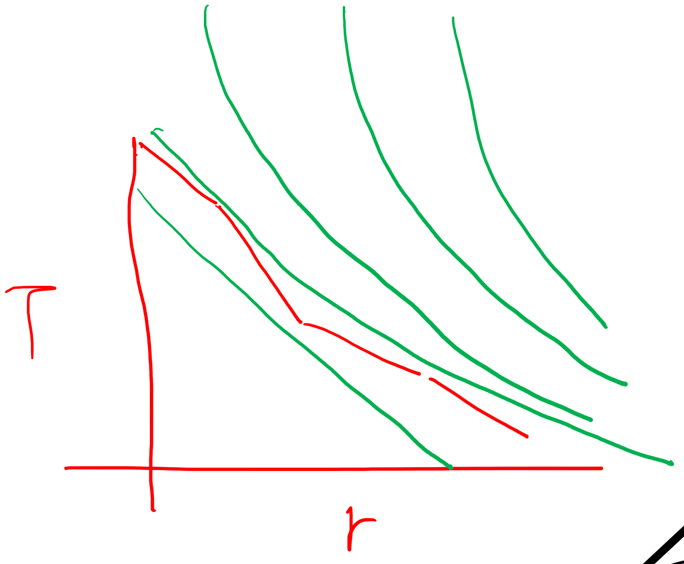
$$\frac{(T(r) - T_1)}{(T_2 - T_1)} = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}$$

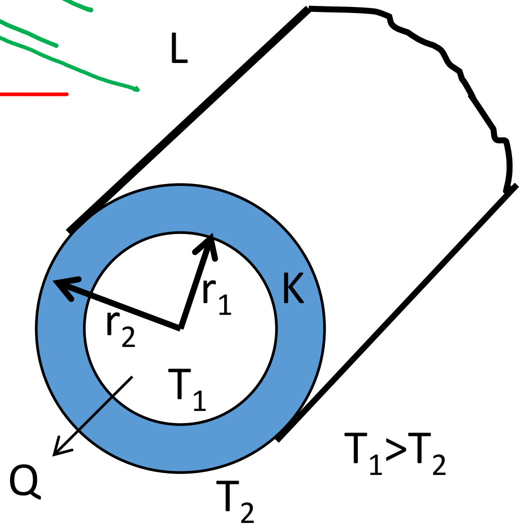


Heat Conduction through Hollow Cylinder

$T = C_1 . \ln r + C_2$ ....(2)

$$C_1 = \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} \quad and \quad C_2 = T_1 - \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} . \ln r_1$$





Substituting values of C_1 & C_2 in Eqn ....(2); We have

$$T = \frac{\ln \frac{r}{r_1}}{\ln \frac{r_2}{r_1}} (T_2 - T_1) + T_1 \quad OR \quad \frac{T - T_1}{T_2 - T_1} = \frac{\ln \frac{r}{r_1}}{\ln \frac{r_2}{r_1}}$$

Heat Conduction through Hollow Cylinder

Heat Flow Rate:

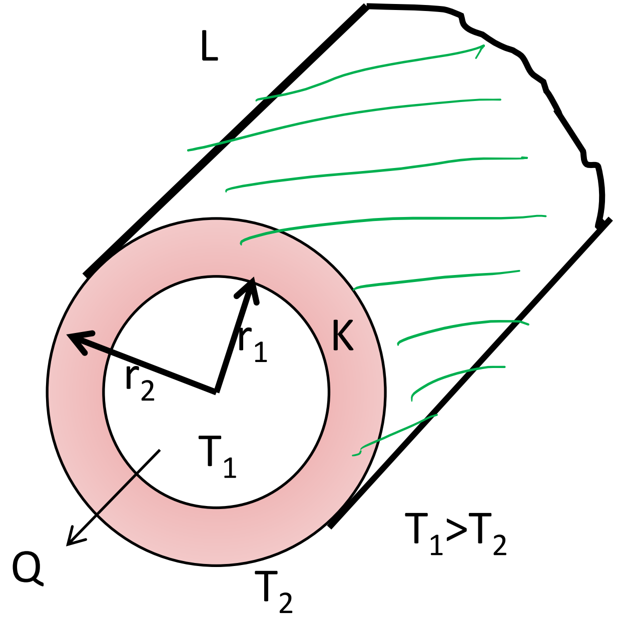
$$Q = -kA\frac{dT}{dr} \qquad \frac{dT}{dr} = \frac{C_1}{r} \text{.... from Eqn....(1)}$$

$$C_1 = \frac{T_2 - T_1}{\ln\frac{r_2}{r_1}}$$

$$\text{Therefore, } Q = -k.2\pi rL.\frac{C_1}{r} = -2\pi kLC_1$$

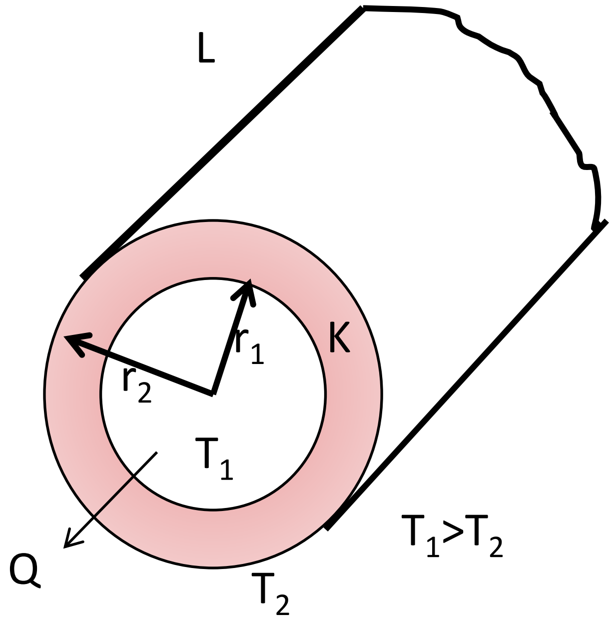
Substituting C_1;

$$Q = -2\pi kL.\frac{(T_2 - T_1)}{\ln\frac{r_2}{r_1}} = 2\pi kL.\frac{(T_1 - T_2)}{\ln\frac{r_2}{r_1}}$$



Logarithmic Mean Area (LMA)

- In case of cylinder, in Q expression, Q=-kA(dT/dr); area transferring heat A=2πrL changes with r, unlike in case of slab. Therefore, it is convenient to work out Mean Area A_m for use in analogous formula for slab Q=kA(ΔT/Δx)



$$If\ we\ write\ Q = \frac{2\pi kL.\Delta T}{\ln\frac{r_2}{r_1}} = k.A_m.\frac{\Delta T}{r_2 - r_1}$$

Cylinder

$$Then\ A_m\ is\ mean\ area\ which\ can\ be\ utilized$$
$$in\ formula\ for\ slab$$

$$A_m = \frac{2\pi L(r_2 - r_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$

m^2

Logarithmic Mean Area (LMA)

$$To\ obtain\ value\ of\ LMA\ i,e.\ A_m;$$

$$We\ multiply\ \&\ divide\ Q\ expression\ by\ (r_2 - r_1)\ as;$$

$$Q = \frac{2\pi kL.\Delta T}{\ln\frac{r_2}{r_1}} \cdot \frac{(r_2 - r_1)}{(r_2 - r_1)} = \frac{k.2\pi L(r_2 - r_1).}{\ln\frac{r_2}{r_1}} \frac{\Delta T}{(r_2 - r_1)}$$

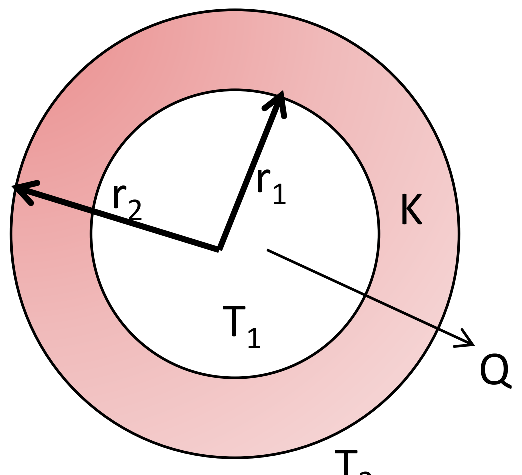
$$Comparing\ with\ Q = k.A_m.\frac{\Delta T}{r_2 - r_1};$$

$$We\ have\quad A_m = \frac{2\pi L(r_2 - r_1)}{\ln\frac{r_2}{r_1}} = \frac{A_o - A_i}{\ln\frac{A_o}{A_i}}$$

One Dimensional (Radial) Steady State Heat Conduction through Hollow Sphere

Consider a hollow sphere of inner radius r_1 and outer r_2 of a material having conductivity k.

Inner surface of sphere is at temp T_1 and outer at T_2



Conduction Equation for one dimensional (radial) Heat flow (without g) will be:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \Rightarrow \frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$

no θ no φ & no dT/dt = 0 (steady)
no g

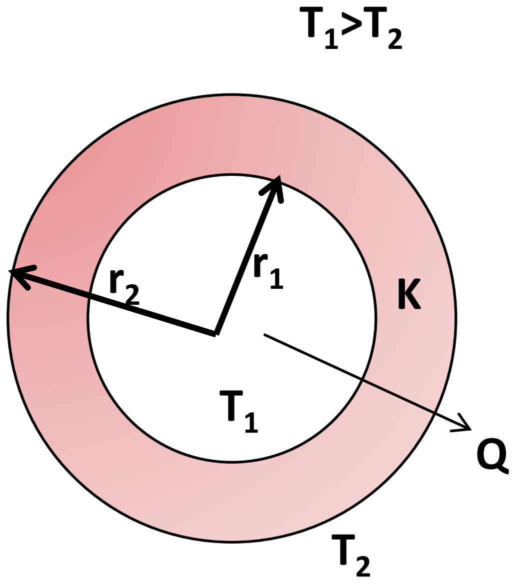
Integrate it twice

One Dimensional (Radial) Steady State Heat Conduction through Hollow Sphere

$$Integrating\ Eqn\ldots\int \frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

$$We\ have\ r^2\frac{dT}{dr} = C_1\ or\ \frac{dT}{dr} = \frac{C_1}{r^2}\ldots(1)$$

Slope of eqn



On further Integration, we have

$$T = -\frac{C_1}{r} + C_2\ldots\ldots\ldots\ldots(2)$$

Temp. variation with radius

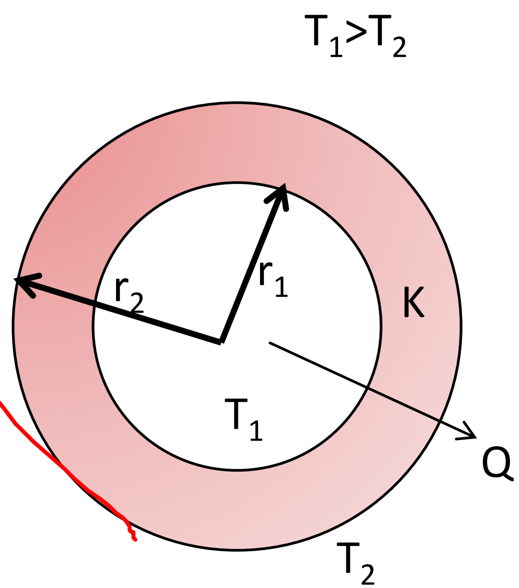
BC's ⇒ ① $r = r_1$ $T = T_1$

② $r = r_2$ $T = T_2$

One Dimensional (Radial) Steady State Heat Conduction through Hollow Sphere

Boundary Conditions:

1) At $r=r_1$; $T=T_1$
2) At $r=r_2$; $T=T_2$



Substituting in Eqn $T = -\frac{C_1}{r} + C_2 \ldots (2)$

$$We\ have\quad C_1 = \frac{(T_1 - T_2).r_1 r_2}{r_1 - r_2}$$

$$And\quad C_2 = T_1 + \frac{(T_1 - T_2).r_2}{r_1 - r_2}$$

$$T = -\frac{(T_1 - T_2) r_1 r_2}{r(r_1 - r_2)} + T_1 + \frac{(T_1 - T_2)}{(r_1 - r_2)} r_2$$

$$Substituting\ C_1\ \&\ C_2\ in\ Eqn\ T = -\frac{C_1}{r} + C_2$$

$$T = \frac{r_1}{r}.\frac{r_2 - r}{r_2 - r_1}.T_1 + \frac{r_2}{r}.\frac{r - r_1}{r_2 - r_1}.T_2$$

This is the Temp Profile across the thickness of sphere

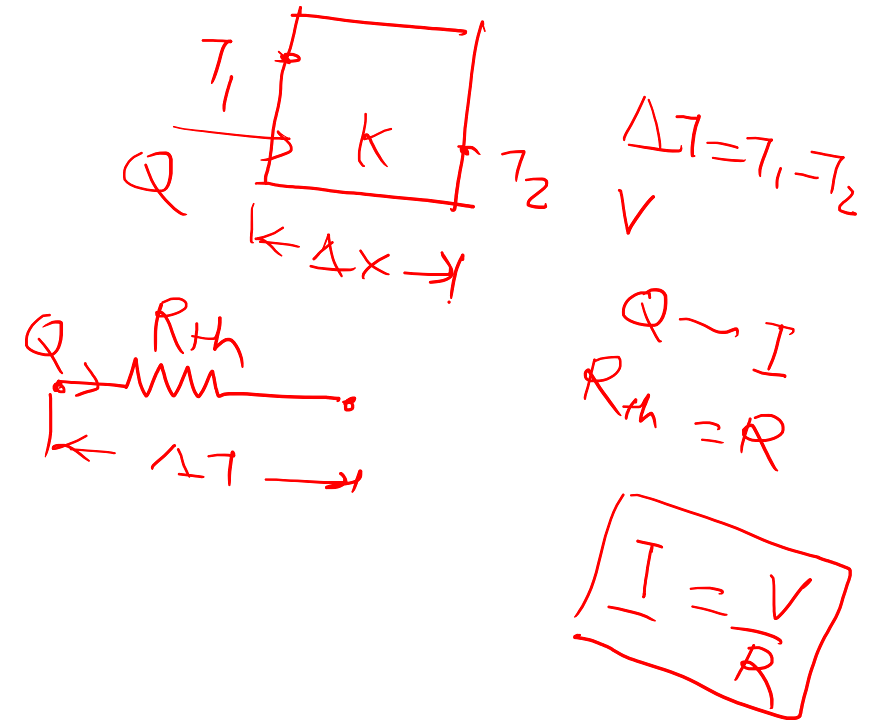
$$Heat\ Flow\ Rate\ Q = -kA\frac{dT}{dr} = -k.4\pi r^2.\frac{dT}{dr}$$

$$Substituting\ \frac{dT}{dr} = \frac{C_1}{r} \Rightarrow Q = -k.4\pi r^2.\frac{C_1}{r^2} = -4\pi k C_1$$

$$Substituting\ C_1;$$

$$Q = 4\pi k.r_2 r_1.\frac{T_1 - T_2}{r_2 - r_1} = \frac{T_1 - T_2}{\dfrac{r_2 - r_1}{4\pi k.r_2 r_1}}$$

$$Therefore\quad R_{cond} = \frac{r_2 - r_1}{4\pi k r_2 r_1}\ and\ A_m = 4\pi r_1 r_2$$



Electrical Analogy

Overall Heat Transfer Coefficient

Heat Flow Rate can also be given as Q=UAΔT;
where U is called as overall heat transfer coefficient

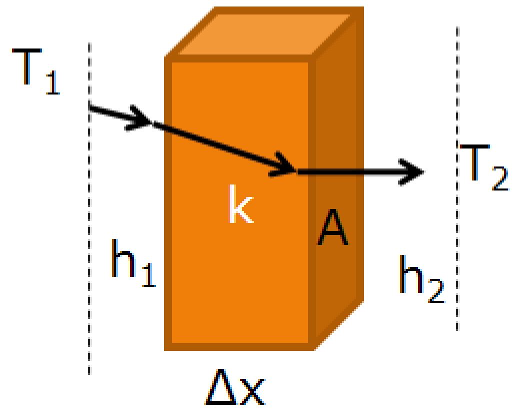
For plane wall:

$$Q = UA\Delta T = \frac{\Delta T}{\frac{1}{UA}} = \frac{\Delta T}{\frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_2 A}}$$

$$hence \quad \frac{1}{UA} = \frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_2 A}$$

$$Therefore, \quad \frac{1}{U} = \frac{1}{h_1} + \frac{\Delta x}{k} + \frac{1}{h_2}$$

$$\cdots where\, U \;\; is\; Overall\; Heat\; Transfer\; Coeff$$

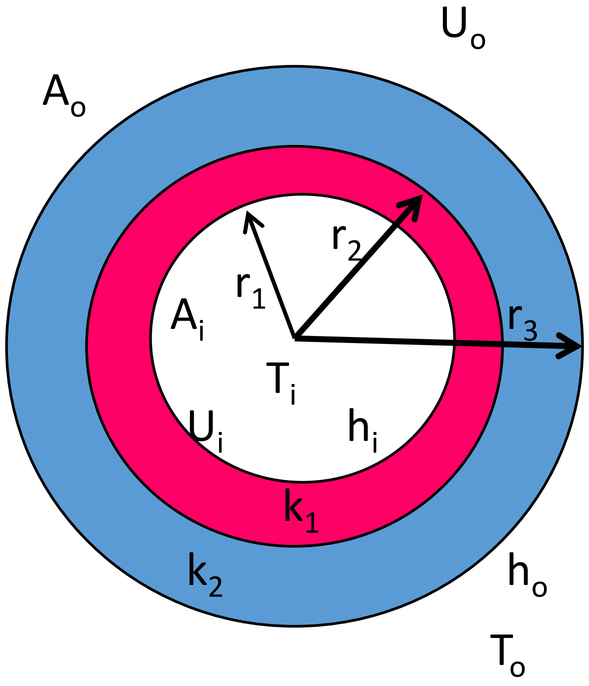


For Cylinder:

$$Q = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o)$$

$$= \frac{\Delta T}{\frac{1}{U_i A_i}} = \frac{\Delta T}{\frac{1}{U_o A_o}} = \frac{\Delta T}{\frac{1}{h_i A_i} + \frac{\ln r_2/r_1}{2\pi k_1 L} + \frac{\ln r_3/r_2}{2\pi k_2 L} + \frac{1}{h_o A_o}}$$

$$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{\ln r_2/r_1}{2\pi k_1 L} + \frac{\ln r_3/r_2}{2\pi k_2 L} + \frac{1}{h_o A_o}$$



where U_i *is Overall heat transfer coeff based on inner surface area* A_i

and U_o *is Overall heat transfer coeff based on outer surface area* A_o

$A_i = 2\pi r_1 L$ *and* $A_o = 2\pi r_3 L$

For Sphere:

$$Q = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o)$$

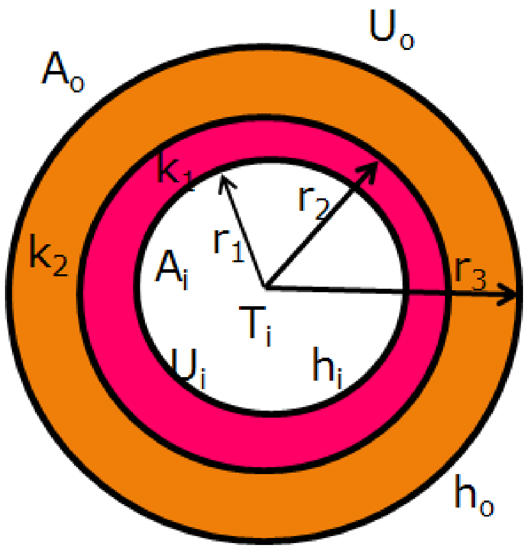
$$= \frac{\Delta T}{\frac{1}{U_i A_i}} = \frac{\Delta T}{\frac{1}{U_o A_o}} = \frac{\Delta T}{\frac{1}{h_i A_i} + \frac{r_2 - r_1}{4\pi k_1 r_2 r_1} + \frac{r_3 - r_2}{4\pi k_2 r_3 r_2} + \frac{1}{h_o A_o}}$$

$$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{r_2 - r_1}{4\pi k_1 r_2 r_1} + \frac{r_3 - r_2}{4\pi k_2 r_3 r_2} + \frac{1}{h_o A_o}$$

where U_i *is Overall heat transfer coeff based on inner surface area* A_i

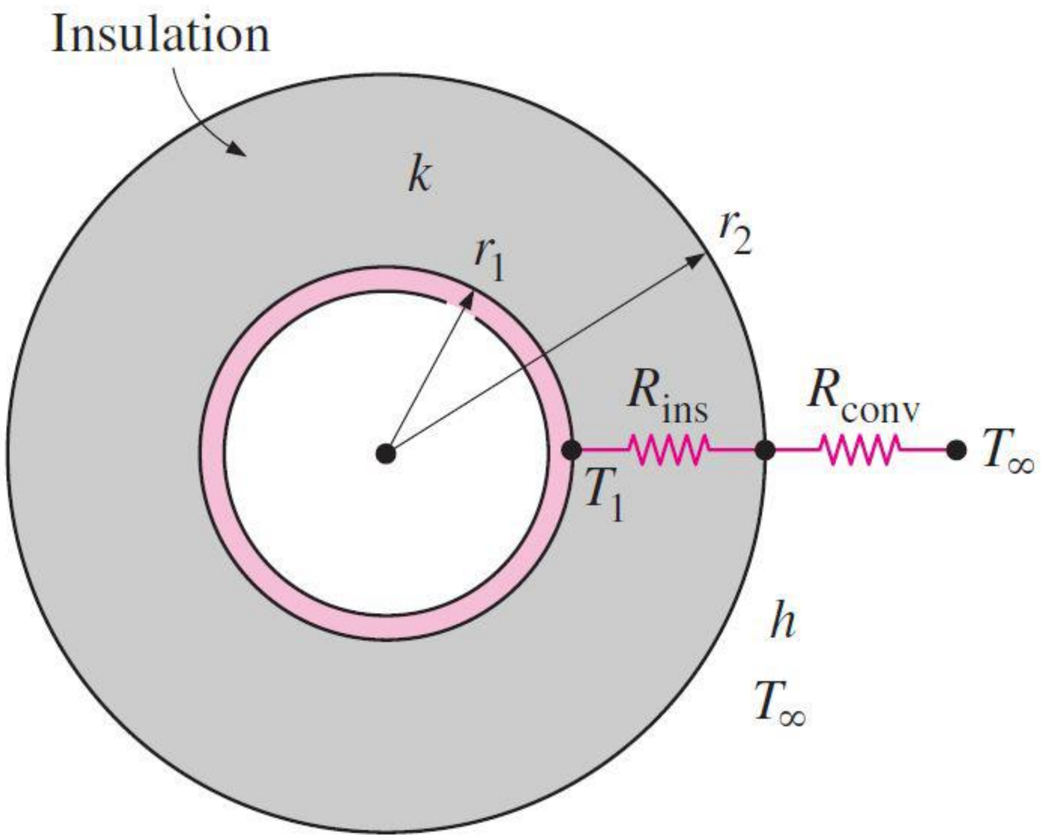
and U_o *is Overall heat transfer coeff based on outer surface area* A_o

$A_i = 4\pi r_1^2$ *and* $A_o = 4\pi r_3^2$



The Critical Radius of Insulation

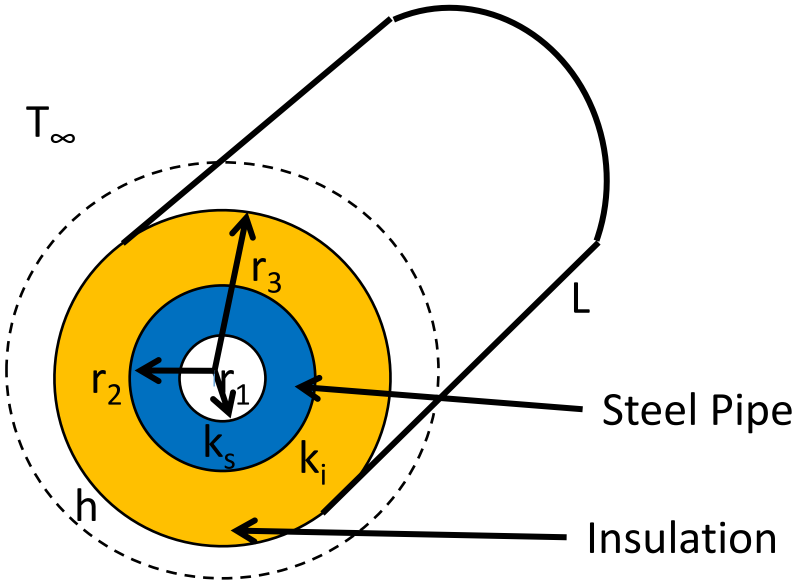
- We know that by adding more insulation to a wall always decreases heat transfer.
- This is expected, since the heat transfer area A is constant, and adding insulation will always increase the thermal resistance of the wall without affecting the convection resistance. However, adding insulation to a cylindrical piece or a spherical shell, is a different matter.
- The additional insulation increases the conduction resistance of the insulation layer but it also decreases the convection resistance of the surface because of the increase in the outer surface area for convection. Therefore, the heat transfer from a pipe may increase or decrease, depending on which effect dominates.



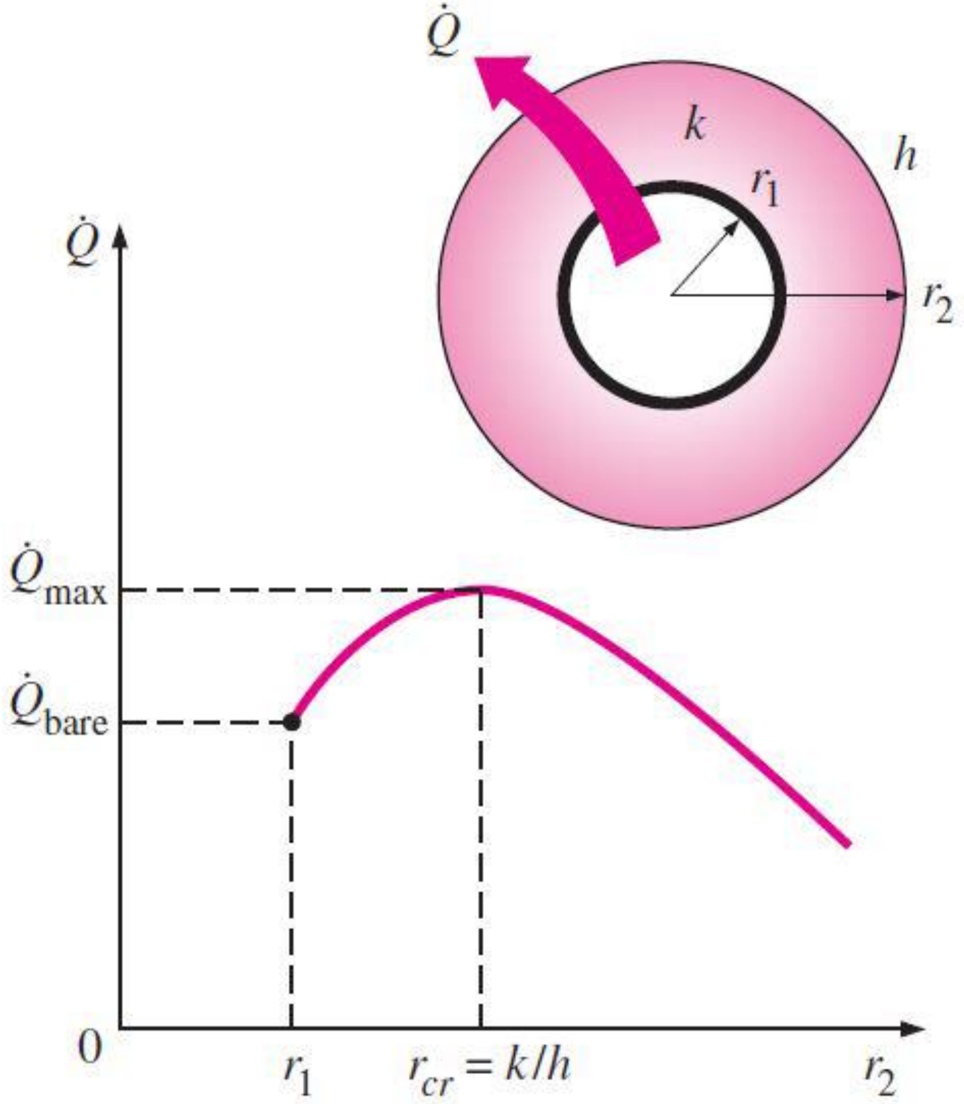
Take the example of heat flow across a steel tube, carrying hot fluid, when a layer of insulation is applied to it as shown in Fig.

Hence;

$$Q = \frac{\Delta T}{\frac{\ln \frac{r_3}{r_2}}{2\pi k_i L} + \frac{1}{h 2\pi r_3 L}}$$

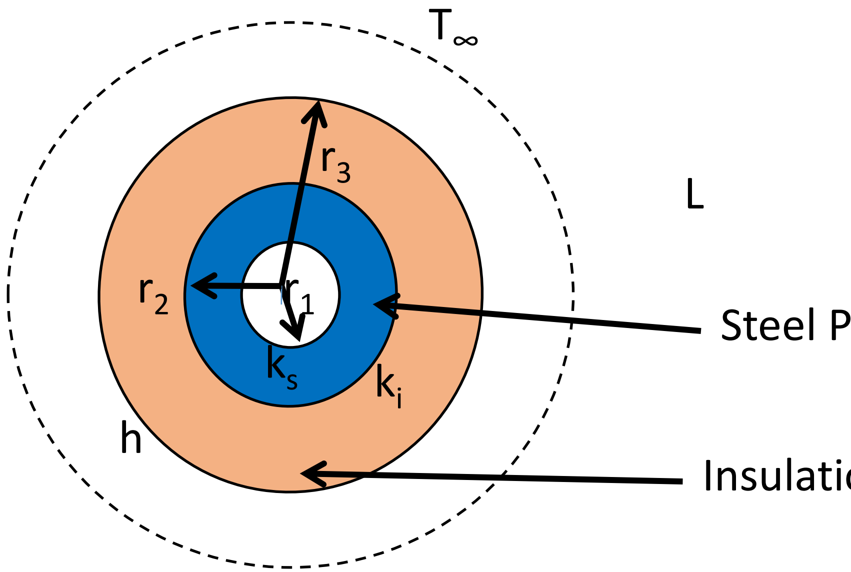


With increase in insulation radius r_3, conductive resistance increases but convective resistance decreases, so we do not know whether Q will increase or decrease.


$\dot{Q}$
k
h
r_1
r_2
$\dot{Q}$
$\dot{Q}_{max}$
$\dot{Q}_{bare}$
0
r_1
$r_{cr} = k/h$
r_2

Critical Radius of Insulation: Sphere

Now, take another example of heat flow across a sphere, having hot fluid, when a layer of insulation is applied to it as shown in Fig.



Hence;

$$Q = \frac{\Delta T}{\frac{r_3 - r_2}{4\pi k_i r_3 r_2} + \frac{1}{h 4\pi r_3^{\,2}}}$$

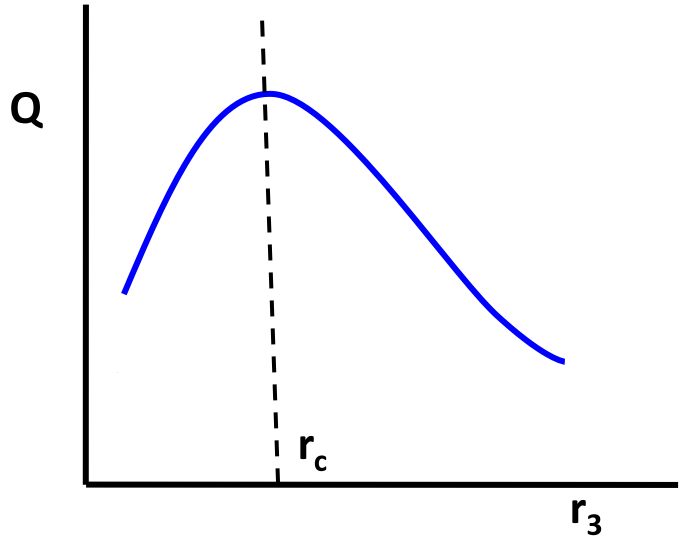
Again, with increase in insulation thickness (r_3) , conductive resistance increases but convective resistance decreases, so we do not know whether Q will increase or decrease.

Critical Radius of Insulation: Sphere/Cylinder

From Q expression, it is found that while conductive resistance increases with r_3 , convective resistance decreases.

It is seen from Q v/s r_3 plot that with increase in r_3 , Q first increases up to certain $r_3 = r_c$, and then starts decreasing.

Value of r_3 , for which Q is max or in other words, total resistance is minimum, is called critical radius of insulation, denoted by r_c



We have to find that value of r_2 , for which Q is maximum or total resistance is minimum.

To obtain maxima, we can either differentiate Q or resistance expression wrt r_2 and put it equal to zero.

$$Therefore we can write:$$

$$\frac{d}{dr_2}\left[\frac{\ln\frac{r_2}{r_1}}{2\pi kL}+\frac{1}{h2\pi r_2 L}\right]=0$$

$$Taking out common we can write:$$

$$\frac{d}{dr_2}\left[\frac{\ln\frac{r_2}{r_1}}{k}+\frac{1}{hr_2}\right]=0$$

$$On\ differentiation: \frac{1}{k}.\frac{1}{r_2} - \frac{1}{h}.\frac{1}{{r_2}^2} = 0$$

$$hence\ r_2 = \frac{k}{h} = r_c$$

$$This\ value\ r_2 = r_c = \frac{k}{h}\ is\ called\ CRITICAL\ \ RADIUS\ \ of\ \ INSULATION$$

$$for\ \ CYLINDER$$

Similarly, it can be shown that critical radius of insulation for sphere is:

$$r_2 = r_c = 2k/h$$

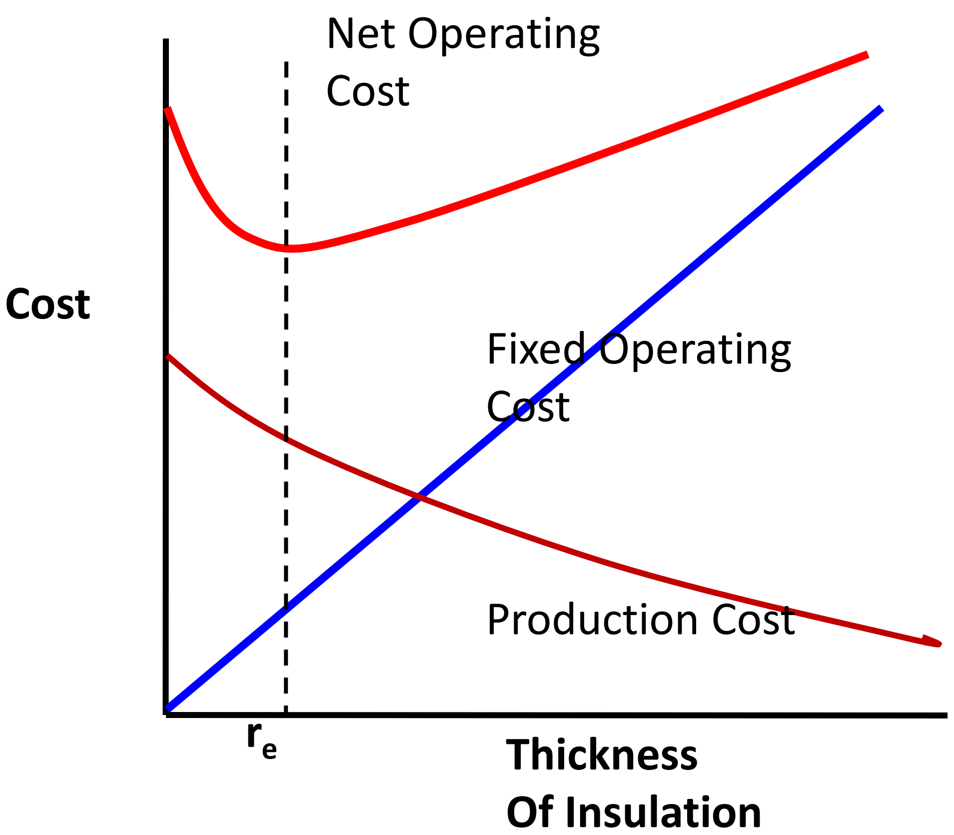
Economic Thickness of Insulation

- Concept based on economics

- As thickness of insulation increases, heat loss decreases, hence production cost decreases .

- However, depreciation & maintenance called fixed operating cost, increases

- Therefore, net operating cost, which is production cost plus fixed operating cost, initially decreases and then increases. The radius (thickness), at which net operating cost is minimum, is known as Economic Radius (Thickness) of Insulation (r_e).



Heat Transfer - Transfer of Heat energy / time (W)

- Conduction → Solids, liquids, gasses
  - electron flow, lattice vibn
  - Fourier's law $Q = -KA\frac{dT}{dx}$
- Convection
- Radiation

Conduction eqn = $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

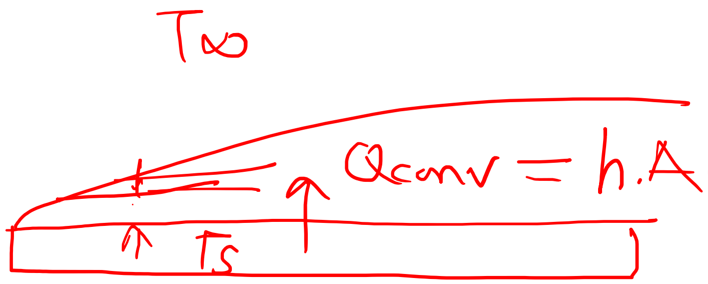
- $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$: Fourier's eqn
- $\frac{q_g}{k}$: without int Heat generation
- $\frac{1}{\alpha}\frac{\partial T}{\partial t}$: 0 Steady state

$$\frac{d^2 T}{dx^2} = 0$$

Laplace form for 1D

Convection :- Conduction + Advection (molecular movement)

Fluids → Heat Exchangers



$Q_{conv} = h \cdot A (T_s - T_\infty)$

Radiation :- Due to flow of elemag. waves — (small energy packet) Photons / quantas

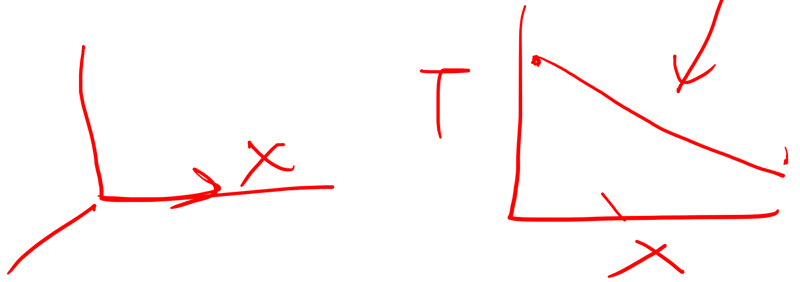
Solar device $Q_{rad} = \epsilon \cdot \sigma A (T_s^4 - T_\infty^4)$

Conduction in steady state without Heat gen. 1D

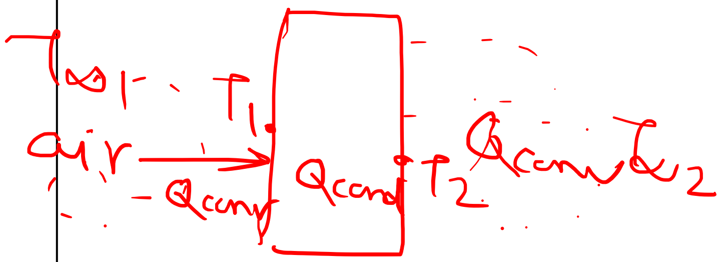
Cartesian $\frac{d^2T}{dx^2} = 0 \Rightarrow T(x)$ & Q (W) q'' (W/m²)

Cyl. $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$

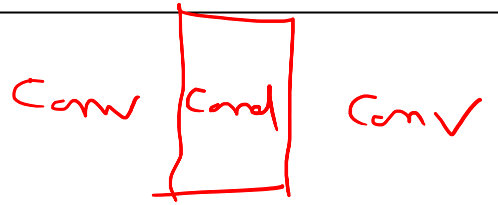
Spherical $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$

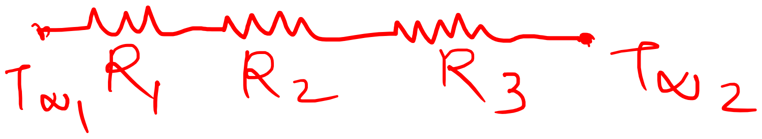


$T_{\infty 1} > T_1 > T_2 > T_{\infty 2}$



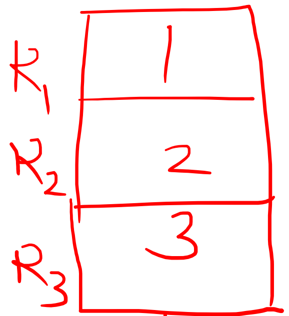
$Q = Q_{conv} + Q_{cond} + Q_{conv}$



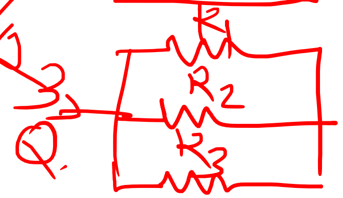


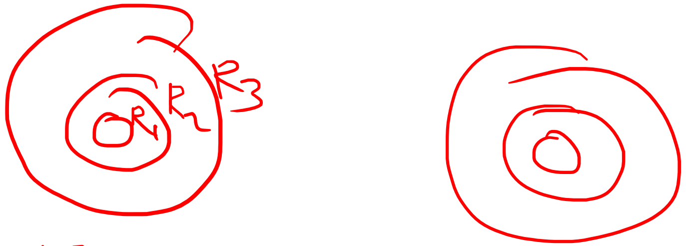
$$Q = \frac{(T_{\infty 1} - T_{\infty 2})}{R_{total}}$$

$$R_{total} = R_1 + R_2 + R_3$$



Parallel $\Rightarrow$ $$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

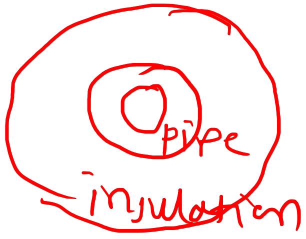


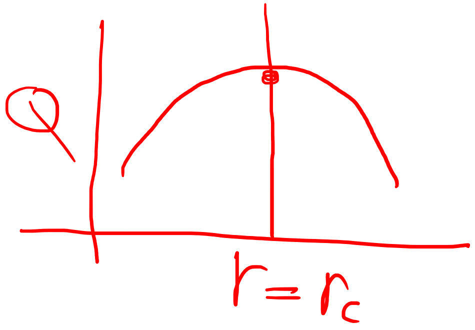


$$= \frac{(T_1 - T_2)}{\frac{\ln r_2/r_1}{2\pi L k_1} + \frac{\ln r_3/r_2}{2\pi L k_2} + \frac{\ln r_4/r_3}{2\pi L k_3} + \frac{1}{h\,2\pi L r_4}}$$

Critical Radius of Insulation

$$Q = \frac{\Delta T}{R_{slab} + R_{ins}} = \frac{\Delta T}{\frac{\Delta x}{kA} + \frac{\Delta b}{k_i A}}$$



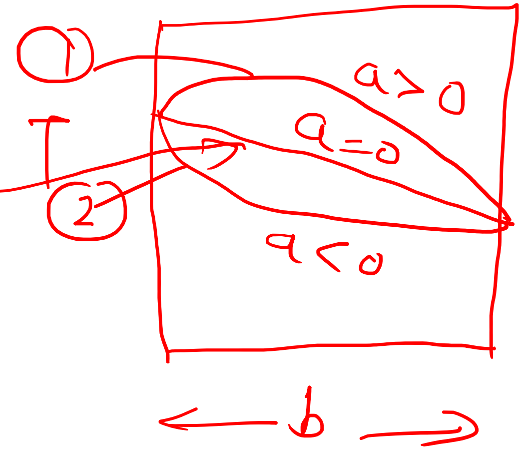


$$r_c = \frac{k}{h} \text{ for cyl.}$$

$$r_c = \frac{2k}{h} \text{ for sphere}$$

Variable conductivity

K = const $\Rightarrow$ $Q = KA\frac{dT}{dx}$



$K = K_0(1 + aT + bT^2 \ldots\ldots)$

Solids $K \propto \frac{1}{T}$ Non metals $K \propto T$

Liquids $K \propto \frac{1}{T}$

Gases $K \propto T$